

קלקולוס 2 מעודכן

פרק 9 - נגזרות חלקיות

תוכן העניינים

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|---------|--------------------------|
| 1 | נגזרות חלקיות מסדר ראשון |
| 2 | נגזרות חלקיות מסדר שני |

ngezorot chalikitot masder rashedon

shealot

בשאלות 1-10 חשבו את הנגזרות החלקיות מסדר ראשון של הפונקציה הנתונה.

$$f(x, y) = x^5 \ln y \quad (2) \qquad f(x, y) = 4x^3 - 3x^2 y^2 + 2x + 3y \quad (1)$$

$$f(x, y) = (x^2 + y^3) \cdot (2x + 3y) \quad (4) \quad \text{. (רק } f_x \text{)} \quad f(x, y) = \frac{x^2 y^4 (\sqrt{y} + 5 \ln y)}{y^2 + 5y + y^y} \quad (3)$$

$$f(x, y) = \sin(xy) \quad (6) \qquad f(x, y) = \frac{x^2 - 3y}{x + y^2} \quad (5)$$

$$f(r, \theta) = r \cos \theta \quad (8) \qquad f(x, y) = \arctan(2x + 3y) \quad (7)$$

$$f(u, v, t) = e^{uv} \sin(ut) \quad (10) \qquad f(x, y, z) = xy^2 z^3 \quad (9)$$

$$\cdot z(x, y) = \ln(\sqrt{x} + \sqrt{y}) \quad (11) \quad \text{נתון}$$

$$\cdot x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = \frac{1}{2} \quad \text{הוכיחו כי}$$

$$\cdot f(x, y, z) = e^x \left(y^2 - \frac{1}{z} \right) \quad (12) \quad \text{נתון}$$

$$\cdot \frac{\partial f}{\partial x} \left(0, -1, \frac{1}{2} \right), \quad \frac{\partial f}{\partial y} \left(0, -1, \frac{1}{2} \right), \quad \frac{\partial f}{\partial z} \left(0, -1, \frac{1}{2} \right) \quad \text{חשבו}$$

הערת סימון

$$f = f(x, y) \Rightarrow f_x = \frac{\partial f}{\partial x} = f_1 ; \quad f_y = \frac{\partial f}{\partial y} = f_2$$

תשובות סופיות

$$f_y = -6x^2y + 3 \quad f_x = 12x^2 - 6xy^2 + 2 \quad (1)$$

$$f_y = \frac{x^5}{y} \quad f_x = 5x^4 \ln y \quad (2)$$

$$f_x = 2x \frac{y^4(\sqrt{y} + 5 \ln y)}{y^2 + 5y + y^y} \quad (3)$$

$$f_y = 6xy^2 + 12y^3 + 3x^2 \quad f_x = 6x^2 + 6xy + 2y^3 \quad (4)$$

$$f_y = \frac{-3x + 3y^2 - 2x^2y}{(x + y^2)^2} \quad f_x = \frac{x^2 + 2xy^2 + 3y}{(x + y^2)^2} \quad (5)$$

$$f_y = \cos(xy) \cdot x \quad f_x = \cos(xy) \cdot y \quad (6)$$

$$f_y = \frac{3}{1 + (2x + 3y)^2} \quad f_x = \frac{2}{1 + (2x + 3y)^2} \quad (7)$$

$$f_\theta = -r \sin \theta \quad f_r = \cos \theta \quad (8)$$

$$f_z = 3xy^2z^2 \quad f_y = 2xyz^3 \quad f_x = y^2z^3 \quad (9)$$

$$f_t = u \cdot e^{uv} \cdot \cos ut \quad f_v = u \cdot e^{uv} \cdot \sin ut \quad f_u = e^{uv} [v \sin ut + t \cos ut] \quad (10)$$

(11) שאלת הוכחה.

$$\frac{\partial f}{\partial x}\left(0, -1, \frac{1}{2}\right) = -1, \quad \frac{\partial f}{\partial y}\left(0, -1, \frac{1}{2}\right) = -2, \quad \frac{\partial f}{\partial z}\left(0, -1, \frac{1}{2}\right) = 4 \quad (12)$$

הערת סימון

$f_x = \frac{\partial f}{\partial x} = f_1 \quad f_y = \frac{\partial f}{\partial y} = f_2$ $f_{xx} = \frac{\partial^2 f}{\partial x^2} = f_{11} \quad f_{yy} = \frac{\partial^2 f}{\partial y^2} = f_{22}$ $f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = f_{12} \quad f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = f_{21}$

נזרות חלקיות מסדר שני

שאלות

בשאלות 1-14 חשבו את כל הנזרות החלקיות עד סדר שני של הפונקציה הנתונה :

$$f(x, y) = 4x^2 - x^2y^2 + 4x + 10y \quad (1)$$

$$f(x, y) = x^4 \ln y \quad (2)$$

$$f(x, y) = x^3 + y^3 - 6xy \quad (3)$$

$$f(x, y) = x^3 + y^3 + 3(1-y)(x+y) \quad (4)$$

$$f(x, y) = xy(x-y) \quad (5)$$

$$f(x, y) = (x-9)(2y-6)(4x-3y+12) \quad (6)$$

$$f(x, y) = e^{xy}(x+y) \quad (7)$$

$$f(x, y) = e^{x+y} (x^2 + y^2) \quad (8)$$

$$f(x, y) = (x^2 + 2y^2) e^{-(x^2+y^2)} \quad (9)$$

$$f(x, y) = \ln(1+x^2+y^2) \quad (10)$$

$$f(x, y) = \ln(x^2 + y^2) \quad (11)$$

$$f(x, y) = \ln(\sqrt[3]{x^2 + y^2}) \quad (12)$$

$$f(x, y) = \sin(10x + 4y) \quad (13)$$

$$f(x, y, z) = xyz \quad (14)$$

15) חשבו $f(x, y) = \ln(xy - x^2 - y^2)$, עבור $f'_{xy}(1,1)$

16) חשבו $f(x, y) = \ln(x^2 + y^2)$, עבור $f'_{xy}(1,1)$

17) חשבו $f(x, y) = \sqrt{x^2 + y^2}$, עבור $f'_{xy}(1,1)$

18) נתנו $f(x, y) = \frac{x^2}{\ln y + x}$
 $\cdot \frac{\partial^2 f}{\partial x^2}(1,e), \quad \frac{\partial^2 f}{\partial y^2}(1,e), \quad \frac{\partial^2 f}{\partial x \partial y}(1,e)$

הערת סימון

$f_x = \frac{\partial f}{\partial x} = f_1$	$f_y = \frac{\partial f}{\partial y} = f_2$
$f = f(x, y) \Rightarrow f_{xx} = \frac{\partial^2 f}{\partial x^2} = f_{11}$	$f_{yy} = \frac{\partial^2 f}{\partial y^2} = f_{22}$
$f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = f_{12}$	$f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = f_{21}$

תשובות סופיות

$$f_y = -2x^2y + 10 \quad f_{xx} = 8 - 2y^2 \quad f_x = 8x - 2xy^2 + 4 \quad (1)$$

$$f_{yx} = -4xy \quad f_{xy} = -4xy \quad f_{yy} = -2x^2$$

$$f_y = \frac{x^4}{y} \quad f_{xx} = 12x^2 \ln y \quad f_x = 4x^3 \ln y \quad (2)$$

$$f_{yx} = \frac{4x^3}{y} \quad f_{xy} = \frac{4x^3}{y} \quad f_{yy} = -\frac{x^4}{y^2}$$

$$f_y = 3y^2 - 6x \quad f_{xx} = 6x \quad f_x = 3x^2 - 6y \quad (3)$$

$$f_{yx} = -6 \quad f_{xy} = 6 \quad f_{yy} = 6y$$

$$f_y = 3y^2 + 3 - 3x - 6y \quad f_{xx} = 6x \quad f_x = 3x^2 + 3 - 3y \quad (4)$$

$$f_{xy} = -3 \quad f_{yy} = 6y - 6$$

$$f_y = x^2 - 2xy \quad f_{xx} = 2y \quad f_x = 2xy - y^2 \quad (5)$$

$$f_{xy} = f_{yx} = 2x - 2y \quad f_{yy} = -2x$$

$$f_x = 2[8xy - 3y^2 \cdot 1 - 24x - 0 + 57y \cdot 1 + 72 + 0 + 0] \quad (6)$$

$$f_y = 2[4x^2 \cdot 1 - 3x \cdot 2y - 0 - 54y + 57x \cdot 1 + 0 + 27 + 0]$$

$$f_{yy} = 2[0 - 6x \cdot 1 - 54 + 0 + 0] \quad f_{xx} = 2[8y - 0 - 24]$$

$$f_{xy} = 2[8x \cdot 1 - 6y - 0 + 57 + 0]$$

$$f_y = e^{xy} (x^2 + xy + 1) \quad f_x = e^{xy} (xy + y^2 + 1) \quad (7)$$

$$f_{yy} = e^{xy} \cdot x(x^2 + xy + 1) + (0 + x) \cdot e^{xy} \quad f_{xx} = e^{xy} \cdot y(xy + y^2 + 1) + (y + 0 + 0) \cdot e^{xy}$$

$$f_{xy} = e^{xy} \cdot x(xy + y^2 + 1) + (x + 2y) \cdot e^{xy}$$

$$f_y = e^{x+y} (x^2 + y^2 + 2y) \quad f_x = e^{x+y} (x^2 + y^2 + 2x) \quad (8)$$

$$, f_{xx} = e^{x+y} (x^2 + y^2 + 2x) + (2x + 2)e^{x+y}$$

$$f_{yy} = e^{x+y} (x^2 + y^2 + 2y) + (2y + 2)e^{x+y}$$

$$f_{xy} = e^{x+y} (x^2 + y^2 + 2x) + 2y \cdot e^{x+y}$$

$$f_y = e^{-x^2-y^2} (4y - 2x^2y - 4y^3) \quad f_x = e^{-x^2-y^2} (2x - 2x^3 - 4xy^2) \quad (9)$$

$$f_{xx} = e^{-x^2-y^2} (-2x)(2x - 2x^3 - 4xy^2) + (2 - 6x^2 - 4y^2)e^{-x^2-y^2}$$

$$f_{yy} = e^{-x^2-y^2} (-2y)(4y - 2x^2y - 4y^3) + (4 - 2x^2 - 12y^2)e^{-x^2-y^2}$$

$$f_{xy} = e^{-x^2-y^2} (-2y)(2x - 2x^3 - 4xy^2) + (-4x \cdot 2y)e^{-x^2-y^2}$$

$$f_y = \frac{2y}{1+x^2+y^2}$$

$$f_x = \frac{2x}{1+x^2+y^2} \quad (10)$$

$$f_{yy} = \frac{2 \cdot (1+x^2+y^2) - 2y \cdot 2y}{(1+x^2+y^2)}$$

$$f_{xy} = \frac{2y \cdot 2x}{(1+x^2+y^2)^2}$$

$$f_{xx} = \frac{2(x^2+y^2) - 2x \cdot 2x}{(x^2+y^2)^2}$$

$$f_y = \frac{2y}{x^2+y^2}$$

$$f_x = \frac{2x}{x^2+y^2} \quad (11)$$

$$f_{xy} = \frac{0(x^2+y^2) - 2y \cdot 2x}{(x^2+y^2)^2}$$

$$f_{yy} = \frac{2(x^2+y^2) - 2y \cdot 2y}{(x^2+y^2)^2}$$

$$f_{xx} = \frac{2(x^2+y^2) - 2x \cdot 2x}{(x^2+y^2)^2} \cdot \frac{1}{3}$$

$$f_y = \frac{2y}{x^2+y^2} \cdot \frac{1}{3}$$

$$f_x = \frac{2x}{x^2+y^2} \cdot \frac{1}{3} \quad (12)$$

$$f_{xy} = \frac{0(x^2+y^2) - 2y \cdot 2x}{(x^2+y^2)^2} \cdot \frac{1}{3}$$

$$f_{yy} = \frac{2(x^2+y^2) - 2y \cdot 2y}{(x^2+y^2)^2} \cdot \frac{1}{3}$$

$$f_{xx} = -100 \sin(10x+4y)$$

$$f_x = 10 \cos(10x+4y) \quad (13)$$

$$f_{yy} = -16 \sin(10x+4y)$$

$$f_y = 4 \cos(10x+4y)$$

$$f_{yx} = -40 \sin(10x+4y)$$

$$f_{xy} = -40 \sin(10x+4y)$$

$$f_{xz} = y \quad f_{xy} = z$$

$$f_{xx} = 0 \quad f_x = yz \quad (14)$$

$$f_{yz} = x \quad f_{yy} = 0$$

$$f_{yx} = z \quad f_y = xz$$

$$f_{zz} = 0 \quad f_{zy} = x$$

$$f_{zx} = y \quad f_z = xy$$

$$-2 \quad (15)$$

$$-1 \quad (16)$$

$$-\frac{1}{2\sqrt{2}} \quad (17)$$

$$\frac{4}{e^2} \left(1 + \frac{1}{e} \right) \quad (18)$$

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